

# Mark Scheme 4725 January 2006

## Mark Total

1.	(i) $\frac{2 + 16i - i - 8i^2}{10 + 15i}$ (ii) $\frac{1}{5}(10 + 15i)$ or $2 + 3i$	M1 A1 M1 A1  A1ft	2 Attempt to multiply correctly Obtain correct answer Multiply numerator & denominator by conjugate Obtain denominator 5  3 Their part (i) or $10 + 15i$ derived again / 5  <b>5</b>
2.	$1^2 = \frac{1}{6} \times 1 \times 2 \times 3$ $\frac{1}{6}n(n+1)(2n+1) + (n+1)^2$ $\frac{1}{6}(n+1)(n+2)\{2(n+1)+1\}$	B1  M1 DM1  A1 A1	Show result true for $n = 1$ or $2$  Add next term to given sum formula, any letter OK Attempt to factorise or expand and simplify  Correct expression obtained  Specific statement of induction conclusion, with no errors seen  <b>5</b> <b>5</b>
3.	(i) $2 \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1 \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ $2 \times 5 - 1 \times 2 + 3 \times -1$ 5 (ii)	M1  A1 A1 B1ft	Show correct expansion process, allow sign slips  Obtain correct (unsimplified) expression Obtain correct answer State that <b>M</b> is non-singular as $\det \mathbf{M}$ non-zero, ft their determinant  <b>3</b> <b>1</b> <b>4</b>
4.	$u^2 + 4u + 4$ $u^3 + 6u^2 + 12u + 8$  $u = \sqrt[3]{5}$ $x = 2 + \sqrt[3]{5}$	B1  M1 A1 A1ft A1ft	$u + 2$ squared and cubed correctly  Substitute these and attempt to simplify Obtain $u^3 - 5 = 0$ or equivalent  Correct solution to their equation  Obtain 2 + their answer [Decimals score 0/2 of final A marks]  <b>5</b> <b>5</b>

5.	$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1	6	Consider the sum of three separate terms
	$8\Sigma r^3 = 2n^2(n+1)^2$	A1		Correct formula stated or used a.e.f.
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated or used a.e.f.
	$2\Sigma r = n(n+1)$	A1		Correct term seen
	$2n^3(n+1)$	M1		Attempt to factorise or expand and simplify
	<b>AG</b>	A1		Obtain given answer correctly

6.	(i) $\frac{1}{2} \begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}$	B1	7	Transpose leading diagonal and negate other diagonal	
	(ii) Either	B1		Divide by determinant	
	$\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1A1		State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	
	Or	M1		Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$ , or equivalent matrix algebra	
	$\frac{1}{5} \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$	A1ft		5	Matrix multn., two elements correct, for any pair
	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$	B1		All elements correct ft their (i)	
	$\mathbf{B} = \frac{1}{5} \begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$	M1		Find $\mathbf{A}^{-1}$	
	$\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1		Premultiply by $\mathbf{A}^{-1}$ stated or implied	
	Or	A1ft		Matrix multn. Two elements correct	
	$\mathbf{AB} = \begin{pmatrix} 2a+c & 2b+d \\ a+3c & b+3d \end{pmatrix}$	A1		All elements correct	
	$a=0, c=1, b=-0.4, d=2.8$	B1		Correct $\mathbf{B}^{-1}$	
	$\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1		Find $\mathbf{AB}$	
	A1A1	Solve one pair of simultaneous equations			
	A1	Each pair of answers			
		Correct $\mathbf{B}^{-1}$			

7.	<p>(a) (i) <math>\sqrt{13}</math></p> <p>(ii)</p> <p>- 0.59</p> <p>(b)</p> <p><math>1 - 2i</math></p> <p>(c)</p>	<p>B1</p> <p>M1 A1 A1</p> <p>M1</p> <p>A1A1 A1</p> <p>B1 B1</p>	<p>1</p> <p>3</p> <p>4</p> <p>2</p> <p><b>10</b></p>	<p>Obtain correct answer, decimals OK</p> <p>Using <math>\tan^{-1}b/a</math>, or equivalent trig allow + or - Obtain 0.59</p> <p>Obtain correct answer</p> <p>Express LHS in Cartesian form &amp; equate real and imaginary parts Obtain <math>x = 1</math> and <math>y = -2</math></p> <p>Correct answer written as a complex number</p> <p>Sketch of vertical straight line Through <math>(-0.5, 0)</math></p>
8.	<p>(i)</p> <p><math>\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix}</math></p> <p>(ii) Either <math>\begin{pmatrix} 1 &amp; 0 \\ 0 &amp; -1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} 2 &amp; 0 \\ 0 &amp; 2 \end{pmatrix}</math></p> <p>Or <math>\begin{pmatrix} -1 &amp; 0 \\ 0 &amp; 1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} -2 &amp; 0 \\ 0 &amp; -2 \end{pmatrix}</math></p> <p>Or <math>\begin{pmatrix} 2 &amp; 0 \\ 0 &amp; 1 \end{pmatrix}</math></p> <p><math>\begin{pmatrix} 1 &amp; 0 \\ 0 &amp; -2 \end{pmatrix}</math></p>	<p>B1</p> <p>B1 B1</p> <p>B1,B1 B1</p> <p>B1,B1 B1</p> <p>B1,B1 B1</p> <p>B1,B1 B1</p> <p>B1,B1 B1</p>	<p>3</p> <p>6</p> <p><b>9</b></p>	<p>For correct vertex <math>(2, -2)</math></p> <p>For all vertices correct For correct diagram</p> <p>Reflection, in x-axis Correct matrix</p> <p>Enlargement, centre O s.f.2 Correct matrix</p> <p>Reflection, in the y-axis Correct matrix</p> <p>Enlargement, centre O s.f. -2 Correct matrix</p> <p>Stretch, in x-direction s.f. 2 Correct matrix</p> <p>Stretch, in y-direction s.f. -2 Correct matrix</p>

9.	<p>(i) <math>\frac{r+2-r}{r(r+2)}</math> <math>\frac{2}{r(r+2)}</math></p> <p>(ii)</p> <p style="text-align: center;"><b>AG</b></p> $\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$ <p>(iii) (a)</p> $\frac{3}{2}$ <p>(b)</p> $\frac{1}{n+1} + \frac{1}{n+2}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1ft</p> <p>M1</p> <p>A1 ft</p>	<p></p> <p>2</p> <p></p> <p></p> <p>5</p> <p>1</p> <p>2</p> <p><b>10</b></p>	<p>Show correct process for subtracting fractions</p> <p>Obtain given answer correctly</p> <p>Express terms as differences using (i)</p> <p>Express 1<sup>st</sup> 3 (or last 3) terms so that cancelling occurs</p> <p>Obtain <math>1 + \frac{1}{2}</math></p> <p>Obtain <math>-\frac{1}{n+2}, -\frac{1}{n+1}</math></p> <p>Obtain correct answer in any form</p> <p>Obtain value from their sum to <math>n</math> terms</p> <p>Using (iii) (a) – (ii) or method of differences again [ <math>n \rightarrow \infty</math> is a method error ]</p> <p>Obtain answer in any form</p>
10.	<p>(i)</p> $\alpha + \beta + \gamma = 9$ <p>(ii)</p> $p = \frac{9 - \alpha}{2}$ <p>(iii) <math>\alpha\beta\gamma = 29</math></p> <p>(iv)</p> $\alpha(p^2 + q^2) = 29$ $q = \sqrt{\frac{29}{\alpha} - \frac{(9 - \alpha)^2}{4}}$ <p>(iv) Alternative method</p> $2p\alpha + p^2 + q^2 = 27$ $q = \sqrt{27 - \frac{(9 - \alpha)^2}{4} - \alpha(9 - \alpha)}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>1</p> <p>4</p> <p>1</p> <p>5</p> <p><b>11</b></p>	<p>State or use other root is <math>p - iq</math></p> <p>Substitute into (i)</p> <p>Obtain <math>2p + \alpha = 9</math></p> <p>Obtain correct answer a.e.f.</p> <p>Substitute into (iii)</p> <p>Obtain unsimplified expression with no <math>i</math>'s</p> <p>Rearrange to obtain <math>q</math> or <math>q^2</math></p> <p>Substitute their expression for <math>p</math> a.e.f.</p> <p>Obtain correct answer a.e.f.</p> <p>Substitute into <math>\alpha\beta + \beta\gamma + \gamma\alpha = 27</math></p> <p>Obtain unsimplified expression with no <math>i</math>'s</p> <p>Rearrange to obtain <math>q</math> or <math>q^2</math></p> <p>Substitute their expression for <math>p</math> a.e.f.</p> <p>Obtain correct answer a.e.f.</p>